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records of geomagnetic reversal Assessing the fidelity of palaeomagnetic

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Bradford M. Clement

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Assessing the fidelity of palaeomagnetic
records of geomagnetic reversal essing the fidelity of palaeomagnet
records of geomagnetic reversal **records of geomagnetic reversal**
BY BRADFORD M. CLEMENT

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 Mum , PL 33133, ODA (clemento student)
A major difficulty facing the study of geomagnetic polarity reversals lies in interpret-
ing palaeomagnetic records of polarity transitions. These records are the sole source A major difficulty facing the study of geomagnetic polarity reversals lies in interpret-
ing palaeomagnetic records of polarity transitions. These records are the sole source
of information about what happens to the field A major difficulty facing the study of geomagnetic polarity reversals lies in interpret-
ing palaeomagnetic records of polarity transitions. These records are the sole source
of information about what happens to the field ing palaeomagnetic records of polarity transitions. These records are the sole source
of information about what happens to the field as it reverses. In addition to com-
parison of records of the same reversal from differen of information about what happens to the field as it reverses. In addition to com-
parison of records of the same reversal from different types of recorders to check for
accuracy, we need some internal measure that can be parison of records of the same reversal from different types of recorders to check for accuracy, we need some internal measure that can be used as a gauge of the temporal resolution provided by the data. This paper explore accuracy, we need some internal measure that can be used as a gauge of the temporal
resolution provided by the data. This paper explores methods using the extent to
which secular variation is recorded in the full polarity resolution provided by the data. This paper explores methods using the extent to which secular variation is recorded in the full polarity intervals bounding polarity transitions to estimate the temporal extent to which the which secular variation is recorded in the full polarity intervals bounding polarity
transitions to estimate the temporal extent to which the transitional record should
be interpreted. Cumulative dispersion and two autocor transitions to estimate the temporal extent to which the transitional record should
be interpreted. Cumulative dispersion and two autocorrelation methods are evalu-
ated using datasets representative of different end-membe be interpreted. Cumulative dispersion and two ated using datasets representative of different of secular variation and polarity transitions.

Fraction and polarity transitions.
Keywords: geomagnetic polarity reversals; polarity transitions;
palaeomagnetism: secular variation palaeomagnetism ;
palaeomagnetism ; secular variation

1. Introduction

In recent years, an increasing number of palaeomagnetic records of polarity transitions have become available. These records provide the only source of information In recent years, an increasing number of palaeomagnetic records of polarity transi-
tions have become available. These records provide the only source of information
about the behaviour of the geomagnetic field as it rever tions have become available. These records provide the only source of information
about the behaviour of the geomagnetic field as it reverses, so it is critical to find a
way to assess their fidelity. Because transition re about the behaviour of the geomagnetic field as it reverses, so it is critical to find a
way to assess their fidelity. Because transition records document the field with a wide
range of resolution, it is important to devel

way to assess their fidelity. Because transition records document the field with a wide
range of resolution, it is important to develop a way of evaluating the extent to which
a recorder has succeeded in documenting transi range of resolution, it is important to develop a way of evaluating the extent to which
a recorder has succeeded in documenting transitional field behaviour. This means not
only figuring out the accuracy, but also the temp a recorder has succeeded in documenting transitional field behaviour. This means not
only figuring out the accuracy, but also the temporal resolution. This task is difficult
because each palaeomagnetic recorder filters the only figuring out the accuracy, but also the temporal resolution. This task is difficult
because each palaeomagnetic recorder filters the record in ways that are not well
understood. Perhaps what is more important is that because each palaeomagnetic recorder filters the record in ways that are not well
understood. Perhaps what is more important is that there is not a predictive model
of transitional field behaviour to test a record against, understood. Perhaps what is more important is that there is not a predictive model
of transitional field behaviour to test a record against, unlike records of full polarity
intervals that can be tested against the geocentr In developing basic criteria for assessing transition records, it is probably wise to
In developing basic criteria for assessing transition records, it is probably wise to

intervals that can be tested against the geocentric axial dipole (GAD) hypothesis.
In developing basic criteria for assessing transition records, it is probably wise to make as much use of the predictive power of the GAD h In developing basic criteria for assessing transition records, it is probably wise to make as much use of the predictive power of the GAD hypothesis as possible. The first measure of the accuracy of a polarity transition r make as much use of the predictive power of the GAD hypothesis as possible. The
first measure of the accuracy of a polarity transition record must be the accuracy
with which it records the Earth's magnetic field during ful The first measure of the accuracy of a polarity transition record must be the accuracy with which it records the Earth's magnetic field during full polarity intervals when the field is strong. The full polarity intervals with which it records the Earth's magnetic field during full polarity intervals when
the field is strong. The full polarity intervals bounding a transition record must be
recorded by antipodal directions when averaged over the field is strong. The full polarity intervals bounding a transition record must be
recorded by antipodal directions when averaged over appropriate time-scales. Only
after it is shown that the material accurately records recorded by antipodal directions when averaged over appropriate time-scales. Only after it is shown that the material accurately records the field when it is strong should the transitional directions, recorded when the fie interpreted.

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In this paper, this philosophy is taken a step further by using the capability of In this paper, this philosophy is taken a step further by using the capability of a recorder to record secular variation during full polarity intervals as a measure of the temporal resolution that the transition record pr In this paper, this philosophy is taken a step further by using the capability of
a recorder to record secular variation during full polarity intervals as a measure of
the temporal resolution that the transition record pro the temporal resolution that the transition record provides. Again, this is difficult to determine because we do not know how the field was varying during the full polarity intervals, and we do not know the filtering effect that the recorder superimposes on the record. It is possible, however, to make a few assumptions that help make this problem tractable. the record. It is possible, however, to make a few assumptions that help make this e record. It is possible, however, to make a few assumptions that help make this
oblem tractable.
Palaeomagnetists have long used the dispersion of directions recorded at a site to
d out if secular variation has been avera

problem tractable.
Palaeomagnetists have long used the dispersion of directions recorded at a site to
find out if secular variation has been averaged out, leaving behind a mean direction
that records the average GAD direct Palaeomagnetists have long used the dispersion of directions recorded at a site to find out if secular variation has been averaged out, leaving behind a mean direction that records the average GAD direction. By comparing t find out if secular variation has been averaged out, leaving behind a mean direction
that records the average GAD direction. By comparing the observed amount of dis-
persion with that predicted for the site latitude by glo that records the average GAD direction. By comparing the observed amount of dis-
persion with that predicted for the site latitude by global models of secular variation,
it is possible to gauge how effectively secular vari persion with that predicted for the site latitude by global models of secular variation,
it is possible to gauge how effectively secular variation has been averaged out. This
approach can be extended by determining the str it is possible to gauge how effectively secular variation has been
approach can be extended by determining the stratigraphic int
included to average out secular variation as recorded at a site.
If we assume that typical se approach can be extended by determining the stratigraphic interval that must be
included to average out secular variation as recorded at a site.
If we assume that typical secular variation of the geomagnetic field occurs o

continuum of time-scales, as suggested by historical data (Courtillot & Valet 1995), continuum of time-scales, as suggested by historical data (Courtillot & Valet 1995),
it becomes clear that no palaeomagnetic recorder can capture all the detail. For each
recorder there is likely to be a cutoff beyond whi it becomes clear that no palaeomagnetic recorder can capture all the detail. For each it becomes clear that no palaeomagnetic recorder can capture all the detail. For each
recorder there is likely to be a cutoff beyond which higher-frequency (sediments) or
lower-frequency (lavas) information is not recorded recorder there is likely to be a cutoff beyond which higher-frequency (sediments) or
lower-frequency (lavas) information is not recorded. For sediments this is because
the remanence is thought to be locked-in over a finite $\overline{0}$ lower-frequency (lavas) information is not recorded. For sediments this is because
the remanence is thought to be locked-in over a finite thickness of sediment, and the
geomagnetic field behaviour is integrated over this i geomagnetic field behaviour is integrated over this interval. Lavas probably have a low-frequency cutoff because it is rare to find sequences where flows were erupted at frequent intervals for a geologically long time. If geomagnetic field behaviour is integrated over this interval. Lavas probably have a low-frequency cutoff because it is rare to find sequences where flows were erupted
at frequent intervals for a geologically long time. If it were possible to determine
the cutoff for a record, this would provide an importa at frequent intervals for a geologicative cutoff for a record, this would p resolution provided by the record.
Two different approaches to this the cutoff for a record, this would provide an important constraint on the temporal
resolution provided by the record.
Two different approaches to this problem are discussed in this paper. The first

method is an extension of the traditional analysis of dispersion at a site to include Two different approaches to this problem are discussed in this paper. The first
method is an extension of the traditional analysis of dispersion at a site to include
the temporal aspect. In this method, the dispersion abou method is an extension of the traditional analysis of dispersion at a site to include
the temporal aspect. In this method, the dispersion about the unit-vector mean is
determined as successively more stratigraphic interval the temporal aspect. In this method, the dispersion about the unit-vector mean is
determined as successively more stratigraphic intervals are included in the mean.
The average interval required to obtain the value of dispe determined as successively more stratigraphic intervals are included in the mean.
The average interval required to obtain the value of dispersion that represents the whole section provides a measure of the extent to which France the temperature of the second state of the average of the second state $\frac{$ note section provides a measure of the extent to which secular variation has been
corded.
The second method applies standard autocorrelation processes to a unit-vector
ries. Within this method, two different, previously pu

recorded.
The second method applies standard autocorrelation processes to a unit-vector
series. Within this method, two different, previously published, vector-correlation
techniques are compared. As with standard autocorr The second method applies standard autocorrelation processes to a unit-vector
series. Within this method, two different, previously published, vector-correlation
techniques are compared. As with standard autocorrelation fu series. Within this method, two different, previously published, vector-correlation techniques are compared. As with standard autocorrelation functions, the initial slope of the function provides a measure of the memory of \geq magnetic records, the slope may be used to estimate the average interval over which slope of the function provides a measure of the memory of the system. In palaeo-
magnetic records, the slope may be used to estimate the average interval over which
each direction is dependent on previous directions. Assum magnetic records, the slope may be used to estimate the average interval over which
each direction is dependent on previous directions. Assuming that the field varied
smoothly with time, the memory can be interpreted as a each direction is dependent on previous of smoothly with time, the memory can be temporal resolution of secular variation. temporal resolution of secular variation.
2. Cumulative dispersion

In studies in which it is important that a time-averaged field direction is obtained, In studies in which it is important that a time-averaged field direction is obtained,
for example in palaeomagnetic pole determinations, the magnitude of the dispersion
about the mean is compared with the magnitude predict In studies in which it is important that a time-averaged field direction is obtained,
for example in palaeomagnetic pole determinations, the magnitude of the dispersion
about the mean is compared with the magnitude predic for example in palaeomagnetic pole determinations, the magnitude of the dispersion
about the mean is compared with the magnitude predicted for the site latitude by
global models of secular variation (McElhinny & McFadden about the mean is compared with the magnitude predicted for the site latitude by global models of secular variation (McElhinny $\&$ McFadden 1997; Constable 1990; Vandamme 1994). If the observed dispersion is close to tha

then it can be reasonably argued that enough time has been sampled at the site to then it can be reasonably argued that enough time has been sampled at the site to effectively average out secular variation. Conversely, if the observed dispersion is too small, then it is likely that secular variation has then it can be reasonably argued that enough time has been sampled at effectively average out secular variation. Conversely, if the observed disper small, then it is likely that secular variation has not been averaged out. ectively average out secular variation. Conversely, if the observed dispersion is too
nall, then it is likely that secular variation has not been averaged out.
This approach may be expanded to include the available stratig

small, then it is likely that secular variation has not been averaged out.
This approach may be expanded to include the available stratigraphic information
to determine the interval that must be included in the average, to This approach may be expanded to include the available stratigraphic information
to determine the interval that must be included in the average, to obtain a dispersion
that is representative of the dataset as a whole. That to determine the interval that must be included in the average, to obtain a dispersion
that is representative of the dataset as a whole. That dispersion can also be compared
with the dispersion predicted by global secular that is representative of the dataset as a whole. That dispersion can also be compared with the dispersion predicted by global secular variation models. In this case, both the magnitude of the dispersion as well as the res with the dispersion predicted by global secular variat
the magnitude of the dispersion as well as the resolution
palaeomagnetic recorder provides can be examined.
Given a set of unit vectors with a Fisher distribut Equivalently e in a set of the dispersion as well as the resolution of secular variation that the laeomagnetic recorder provides can be examined.
Given a set of unit vectors with a Fisher distribution, it is possible to c

palaeomagnetic recorder provides can be examined.
Given a set of unit vectors with a Fisher distribution, it is possible to calculate
an estimate, k, of the true dispersion, κ (Fisher 1953). In the method used here,
k Given a set of unit vectors with a Fisher distribution, it is possible to calculate
an estimate, k, of the true dispersion, κ (Fisher 1953). In the method used here,
k is calculated as a function of the stratigraphic an estimate, k, of the true dispersion, κ (Fisher 1953). In the method used here, k is calculated as a function of the stratigraphic interval that is included in the calculation of the mean. This is done by incrementa k is calculated as a function of the stratigraphic interval that is included in the calculation of the mean. This is done by incrementally increasing the stratigraphic section included in the calculation of the mean. Wh calculation of the mean. This is done by incrementally increasing the stratigraphic
section included in the calculation of the mean. When the values of k are plotted
versus the number of samples included in the mean, it section included in the calculation of the mean. When the values of k are plotted versus the number of samples included in the mean, it is possible to determine the stratigraphic thickness that must be averaged over in stratigraphic thickness that must be averaged over in order to obtain the k value of the entire dataset. This then provides an estimate of the characteristic time-scale of σ cumulative dispersion method.
When calculated singly through a portion of a section, this method provides sigsecular variation that is recorded in that record. This approach is referred to as the

cumulative dispersion method.
When calculated singly through a portion of a section, this method provides significant insight into the nature of different intervals of the record. The initial slope
of the plot indicates ho When calculated singly through a portion of a section, this method provides significant insight into the nature of different intervals of the record. The initial slope of the plot indicates how well the successive samples mificant insight into the nature of different intervals of the record. The initial slope
of the plot indicates how well the successive samples are serially correlated with one
another. For example, if k increases rapidly of the plot indicates how well the successive samples are serially correlated with one
another. For example, if k increases rapidly as additional samples are included in
the average, it means that the directions are not another. For example, if k increases rapidly as additional samples are included in the average, it means that the directions are not tightly grouped in stratigraphic order, but exhibit little dispersion about some mean the average, it means that the directions are not tightly grouped in stratigraphic order, but exhibit little dispersion about some mean direction. If the initial slope is negative, then it means that the directions are tig order, but exhibit little dispersion about some mean direction. If the initial slope
is negative, then it means that the directions are tightly grouped in stratigraphic
order and vary systematically about some mean. If th is negative, then it means that the directions are tightly grouped in stratigraphic
order and vary systematically about some mean. If the initial slope is negative and
very steep, quickly reaching the stable value of k , order and vary systematically about some mean. If the initial slope is negative and
very steep, quickly reaching the stable value of k , it indicates that the directions are
not serially correlated and instead they appro very steep, quickly reaching the stable value of k , it indicates that the directions are
not serially correlated and instead they approximate a random sampling about the
mean direction, with no systematic variation with very steep, quickly reaching the stable value of k, it indicates that the directions are
interesting in the mean instead they approximate a random sampling about the
directions direction, with no systematic variation with mean direction, with no systematic variation with stratigraphic position. As different

Although this information is useful, the results are very sensitive to the starting value for the entire record.
Although this information is useful, the results are very sensitive to the starting
position in some records. For this reason, we employed a jackknife approach, in which
the process is repeated Although this information is useful, the results are very sensitive to the starting position in some records. For this reason, we employed a jackknife approach, in which the process is repeated with the starting point pro position in some records. For this reason, we employed a jackknife approach, in which
the process is repeated with the starting point progressing through the record. For
each successive number of samples included in the m The process is repeated with the starting point progressing through the record. For each successive number of samples included in the mean, the average k is calculated along with the variance about that mean k. Plotting t variance provides results that are more representative of the entire section.
This technique of incrementing the starting point through the data sequence Γ ong with the variance about that mean k . Plotting the mean k values and the riance provides results that are more representative of the entire section.
This technique of incrementing the starting point through the dat

variance provides results that are more representative of the entire section.
This technique of incrementing the starting point through the data sequence and then for each incremental offset calculating the average k va This technique of incrementing the starting point through the data sequence and then for each incremental offset calculating the average k value and the variance about that average gives an indication of the stratigraph then for each incremental offset calculating the average k value and the variance
about that average gives an indication of the stratigraphic interval that must be con-
sidered, on average, in order to average out secul about that average gives an indication of the stratigraphic interval that must be considered, on average, in order to average out secular variation. For example, applying this method to a set of directions that varies sin sidered, on average, in order to average out secular variation. For example, applying
this method to a set of directions that varies sinusoidally yields results such as those
shown in figure 1. The initial values of k are this method to a set of directions that varies sinusoidally yields results such as those shown in figure 1. The initial values of k are high because adjacent directions are closely grouped. As additional parts of the sine wave are included, the value of k decreases, reaching a minimum at an interval corre closely grouped. As additional parts of the sine wave are included, the value of k decreases, reaching a minimum at an interval corresponding to one-quarter of the wavelength of the directional variation. The values of decreases, reaching a minimum at an interval corresponding to one-quarter of the

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no. of samples included in average
Figure 1. Cumulative dispersion for a sequence of unit vectors with a constant declination and
sinusoidally varying inclination (a). Comparison of cumulative dispersion plots obtained for Figure 1. Cumulative dispersion for a sequence of unit vectors with a constant declination and
sinusoidally varying inclination (a) . Comparison of cumulative dispersion plots obtained for a
sequence of unit vectors rando sinusoidally varying inclination (a) . Comparison of cumulative dispersion plots obtained for a sequence of unit vectors randomly selected from a Fisher distribution with a specified mean and dispersion (b) . The effects sequence of unit vectors randomly selected from a Fisher distribution with a specified mean and

Plots of the mean k (or variance about that mean), however, do not provide as inchinitial information about the nature of the record as the single sequence calcu-Plots of the mean k (or variance about that mean), however, do not provide as much initial information about the nature of the record as the single sequence calcu-
lations. This can be seen by examining the cumulative d Plots of the mean k (or variance about that mean), however, do not provide as much initial information about the nature of the record as the single sequence calculations. This can be seen by examining the cumulative dis much initial information about the nature of the record as the single sequence calculations. This can be seen by examining the cumulative dispersion plots obtained for a sequence of random directions selected from a Fishe lations. This can be seen by examining the cumulative dispersion plots obtained for
a sequence of random directions selected from a Fisher distribution with a specified
mean direction and dispersion (κ) . The cumulative a sequence of random directions selected from a Fisher distribution with a specified
mean direction and dispersion (κ) . The cumulative dispersion curve for the random
sequence shows a rapid initial decrease to the stab mean direction and dispersion (κ) . The cumulative dispersion curve for the random
sequence shows a rapid initial decrease to the stable value of k. If the same sequence
is smoothed using a Gaussian filter over 5 and 10 sequence shows a rapid initial decrease to the stable value of k . If the same sequence is smoothed using a Gaussian filter over 5 and 10 points, the resulting curves exhibit progressively shallower slopes (note that lar

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dispersion caused by the smoothing). The dramatic change in k values results from
the effects of the smoothing on the dispersion. dispersion caused by the smoothing). The dram-
the effects of the smoothing on the dispersion.
Although comparing the average k values of spersion caused by the smoothing). The dramatic change in k values results from
e effects of the smoothing on the dispersion.
Although comparing the average k values obtained from the jackknife method
ses some of the

the effects of the smoothing on the dispersion.
Although comparing the average k values obtained from the jackknife method
loses some of the resolution about the serial correlation of the directions over short
intervals loses some of the resolution about the serial correlation of the directions over short intervals, it appears to provide more robust estimates of the characteristic stratigraphic interval of secular variation in that particular record. This means that this intervals, it appears to provide more robust estimates of the characteristic stratigraphic interval of secular variation in that particular record. This means that this method can succeed in providing some information abou graphic interval of secular variation in that particular record. This means that this
method can succeed in providing some information about the temporal resolution of
a sequence of data. As can be seen from figure 1, howe a sequence of data. As can be seen from figure 1, however, the results are not always readily interpreted in terms of the interval that secular variation is averaged over, particularly at higher smoothing levels. readily interpreted in terms of the interval that secular variation is averaged over,

3. Autocorrelation methods

 $\frac{3.4 \text{ّ} \text{Autocorrelation methods}}{3.4 \text{ H}}$
The lack of resolution in the results of the cumulative dispersion method led me
to explore additional approaches to estimating the characteristic interval of secular The lack of resolution in the results of the cumulative dispersion method led me
to explore additional approaches to estimating the characteristic interval of secular
variation as recorded in a particular section. The two The lack of resolution in the results of the cumulative dispersion method led me
to explore additional approaches to estimating the characteristic interval of secular
variation as recorded in a particular section. The two to explore additional approaches to estimating the characteristic interval of secular variation as recorded in a particular section. The two methods described below are based on autocorrelation of a series of unit vectors. dard sequence of scalars is a useful exploratory statistical technique that requires no based on autocorrelation of a series of unit vectors. The autocorrelation of a standard sequence of scalars is a useful exploratory statistical technique that requires no assumptions about the presence of periodicities in dard sequence of scalars is a useful exploratory statistical technique that requires no assumptions about the presence of periodicities in the data. In a standard autocorrelation, a sequence of scalars is compared with its assumptions about the presence of periodicities in the data. In a standard autocor-
relation, a sequence of scalars is compared with itself at increasing offsets or lags.
The sum of the product of each pair of values (minu relation, a sequence of scalars is compared with itself at increasing offsets or lags.
The sum of the product of each pair of values (minus their means) is normalized by
the number of values used in the sum (Priestley 1981

Examining a plot of the autocorrelation coefficient as a function of offset may the number of values used in the sum (Priestley 1981).
Examining a plot of the autocorrelation coefficient as a function of offset may
provide considerable insight into the nature of a sequence. Cyclicity in the data will
 Examining a plot of the autocorrelation coefficient as a function of offset may
provide considerable insight into the nature of a sequence. Cyclicity in the data will
be indicated by a corresponding cyclicity in the correl provide considerable insight into the nature of a sequence. Cyclicity in the data will
be indicated by a corresponding cyclicity in the correlation coefficient. Conversely,
a sequence of random values about a mean will pro be indicated by a corresponding cyclicity in the correlation coefficient. Conversely, a sequence of random values about a mean will produce an autocorrelation function that plummets immediately from a value of 1 to values that plummets immediately from a value of 1 to values varying about 0. If some serial that plummets immediately from a value of 1 to values varying about 0. If some serial correlation exists in the data, then the slope of the autocorrelation function will be more gradual. The initial slope can, in general, correlation exists in the data, then the sk
more gradual. The initial slope can, in gen
memory in the process (Priestley 1981).
By extending this method to a sequence by extending this method to a sequence of unit vectors it may be possible to use the sextending this method to a sequence of unit vectors it may be possible to use the tocorrelation functions in a similar manner to charact

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By extending this method to a sequence of unit vectors it may be possible to use the
autocorrelation functions in a similar manner to characterize the statistical nature
of the seque By extending this method to a sequence of unit vectors it may be possible to use the autocorrelation functions in a similar manner to characterize the statistical nature of the sequence. Ideally, a sequence with true perio autocorrelation functions in a similar manner to characterize the statistical nature
of the sequence. Ideally, a sequence with true periodicity will produce a periodic
autocorrelation function. Likewise, a random sampling of the sequence. Ideally, a sequence with true periodicity will produce a periodic autocorrelation function. Likewise, a random sampling of directions about a mean will produce a very steep initial slope in the autocorrela when direction is not truly independent of the directions about a mean will produce a very steep initial slope in the autocorrelation function. Sequences in
which each direction is not truly independent of the directions immediately above
or below will produce more gentle initial slopes, with which each direction is not truly independent of the directions immediately above
or below will produce more gentle initial slopes, with the slope dependent on how
many samples separate directions that are independent. Thi or below will produce more gentle initial slopes, with the slope dependent on how
many samples separate directions that are independent. This latter case provides
a way of assessing the average stratigraphic interval over many samples separate directions that are independent. This latter case provides a way of assessing the average stratigraphic interval over which the directions are independent. If the directions are recording a smoothly v a way of assessing the average stratigraphic interval over which the directions are independent. If the directions are recording a smoothly varying field on some time-scales, we would expect the directions to be dependent independent. If the directions are recording a scales, we would expect the directions to be de
sequence corresponding to those time-scales. Solution of the scales.
 $\begin{array}{c|c}\n\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet\n\end{array}$ a. Orthogonal transformation method

4. Orthogonal transformation method
Fisher *et al.* (1987) developed a technique for estimating how well one sequence of
unit vectors correlates with another. This method is based on determining how well Fisher *et al.* (1987) developed a technique for estimating how well one sequence of unit vectors correlates with another. This method is based on determining how well one series can be matched by the second by performing Fisher *et al.* (1987) developed a technique for estimating how well one sequence of unit vectors correlates with another. This method is based on determining how well one series can be matched by the second by performing *Phil. Trans. R. Soc. Lond.* A (2000)

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of the second series. If the match is best when the orthogonal transformation is a
rotation, then the association is positive. If the match-up is better when the transof the second series. If the match is best when the orthogonal transformation is a rotation, then the association is positive. If the match-up is better when the trans-formation is a reflection, then the association is neg formation, then the association is positive. If the match-up is better when the trans-
formation is a reflection, then the association is negative. If the first vector series is rotation, then the association is positive. If the match-up is better when the trans-
formation is a reflection, then the association is negative. If the first vector series is
X and the second is X^* , then an estimate formation is a reflection, then the asses X and the second is X^* , then an est calculating the following quantities

$$
S_{XX^*} = \det \left| \sum_{i=1}^n X_i X_i^{*0} \right| = \det \left\{ \sum_{i=1}^n x_i x_i^* \sum_{j} y_i x_i^* \sum_{k} z_i x_i^* \right\},
$$

$$
S_{XX} = \det \left| \sum_{i=1}^n X_i X_i^0 \right|,
$$

$$
S_{XX^*} = \det \left| \sum_{i=1}^n X_i X_i^0 \right|,
$$

$$
S_{X^*X^*} = \det \left| \sum_{i=1}^n X_i^* X_i^* \right|,
$$

where the prime indicates the transpose. The correlation coefficient is given by

$$
L = S_{XX^*}/(S_{XX}S_{X^*X^*})^{1/2}.
$$

This correlation coefficient has the intuitive advantage that if only a pure rotation This correlation coefficient has the intuitive advantage that if only a pure rotation
is required to bring the sequences into agreement, the coefficient has a value of $+1$.
Conversely if a reflection is required then the This correlation coefficient has the intuitive advantage that if only a pure ro
is required to bring the sequences into agreement, the coefficient has a value of
Conversely, if a reflection is required, then the coefficie required to bring the sequences into agreement, the coefficient has a value of $+1$.
myersely, if a reflection is required, then the coefficient has a value of -1 .
This method may be applied to our problem by using it t

Conversely, if a reflection is required, then the coefficient has a value of -1 .
This method may be applied to our problem by using it to calculate the autocor-
relation function for our sequences of unit vectors. We fo This method may be applied to our problem by using it to calculate the autocor-
relation function for our sequences of unit vectors. We follow the standard approach
of comparing the sequence with a successively offset ser relation function for our sequences of unit vectors. We follow the standard approach
of comparing the sequence with a successively offset series of itself, and normalize
by N. The resulting plot provides a measure of the m of comparing the sequence with a successively offset
by N . The resulting plot provides a measure of the n
interval over which the vectors are not independent. interval over which the vectors are not independent.
5. Serial-correlation (dot-product) method

5. **Serial-correlation (dot-product) method**
Watson & Beran (1967) and Epp *et al.* (1971) developed a method of estimating the
serial correlation of unit vectors in a series by comparing the sum of the dot products Watson & Beran (1967) and Epp *et al.* (1971) developed a method of estimating the serial correlation of unit vectors in a series by comparing the sum of the dot products of the pairs of vectors immediately above and imme Watson & Beran (1967) and Epp *et al.* (1971) developed a method of estimating the serial correlation of unit vectors in a series by comparing the sum of the dot products of the pairs of vectors immediately above and imme serial correlation of unit vectors in a series by comparing the sum of the dot products
of the pairs of vectors immediately above and immediately below each sample. This
method can be used specifically to test the hypothes of the pairs of vectors immediately above and immediately below each sample. This method can be used specifically to test the hypothesis that any two successive unit vectors are independent. This approach provides an intui method can be used specifically to test the hypothesis that any two successive unit
vectors are independent. This approach provides an intuitively appealing estimate of
the correlation coefficient in that for two perfectl the correlation coefficient in that for two perfectly aligned unit vectors the coefficient will be $+1$, and for two antipodal vectors, it will be -1 . We have modified this method the correlation coefficient in that for two perfectly aligned unit vectors the coefficient
will be $+1$, and for two antipodal vectors, it will be -1 . We have modified this method
in order to compare a data series with will be $+1$, and for two antipodal vectors, it will be -1 . We have modified this method
in order to compare a data series with itself at increasing offsets, as in calculating a
standard autocorrelation function. In oth in order to compare a data series with itself at increasing standard autocorrelation function. In other words, we sum offset we plot the value of the correlation coefficient L :

$$
L = \sum_{i}^{n-1} X_i X_{i+1}.
$$

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offset
Figure 2. Comparison of (a) the dot-product and (b) the rotation-matrix correlation methods
as autocorrelation tools. The open circles represent results from a sequence of randomly unit Figure 2. Comparison of (a) the dot-product and (b) the rotation-matrix correlation methods
as autocorrelation tools. The open circles represent results from a sequence of randomly unit
vectors drawn from a Eisher distr Figure 2. Comparison of (a) the dot-product and (b) the rotation-matrix correlation methods
as autocorrelation tools. The open circles represent results from a sequence of randomly unit
vectors drawn from a Fisher distrib as autocorrelation tools. The open circles represent results from a sequence of randomly unit
vectors drawn from a Fisher distribution with a known mean and dispersion. The solid circles
and squares show the results obtain respectively. and squares show the results obtained as the random record is smoothed over 5 and 10 points, respectively. $\bf{6.}$ **Examples**

6. Examples
Each of the methods described above was tested using a cyclicly varying sequence of
directions and a sequence of random directions selected from a Fisher distribution Each of the methods described above was tested using a cyclicly varying sequence of directions, and a sequence of random directions selected from a Fisher distribution with a specified mean and κ with different degree Each of the methods described above was tested using a cyclicly varying sequence of directions, and a sequence of random directions selected from a Fisher distribution with a specified mean and κ with different degree directions, and a sequence of random directions selected from a Fisher distribution with a specified mean and κ with different degrees of smoothing applied. We present analysis of two datasets that might be considered th a specified mean and κ with different degrees of smoothing applied. We present alysis of two datasets that might be considered representative of these examples. Just as in an autocorrelation plot of a sequence of s

analysis of two datasets that might be considered representative of these examples.
Just as in an autocorrelation plot of a sequence of scalars, if the sequence of unit
vectors is a series of random variations about a mean Just as in an autocorrelation plot of a sequence of scalars, if the sequence of unit vectors is a series of random variations about a mean, with no serial correlation, then the correlation coefficient will very rapidly fa vectors is a series of random variations about a mean, with no serial correlation, then the correlation coefficient will very rapidly fall from a value of 1 to values that fluctuate about 0. If we consider a series of scal then the correlation coefficient will very rapidly fall from a value of 1 to values that fluctuate about 0. If we consider a series of scalars that define a sine wave, the auto-correlation function will vary systematicall fluctuate about 0. If we consider a series of scalars that define a sine wave, the auto-
correlation function will vary systematically between $+1$ and -1 , with the offset
equal to the wavelength in the scalar series. A correlation function will vary systematically between $+1$ and -1 , with the offset equal to the wavelength in the scalar series. A series of unit vectors that vary sinusoidally, however, will not produce a variation bet soidally, however, will not produce a variation between $+1$ and -1 , because, unlike
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Figure 3. Palaeomagnetic directions obtained from a high-resolution record of recent secular variation obtained from Fish Lake, Oregon (a).

the scalar series, when the directions pass through zero (i.e. when the inclination the scalar series, when the directions pass through zero (i.e. when the inclination are zero, or the directions are horizontal), the vectors still have magnitude equal to 1. Therefore, even when the directional sine waves the scalar series, when the directions pass through zero (i.e. when the inclination
are zero, or the directions are horizontal), the vectors still have magnitude equal
to 1. Therefore, even when the directional sine waves are zero, or the directions are horizontal), the vectors still have magnitude equal
to 1. Therefore, even when the directional sine waves are perfectly out of phase, a
perfectly negative correlation is not obtained, becaus to 1. Therefore, even when the directional sine was
perfectly negative correlation is not obtained, because
zero, two of the vectors are in perfect alignment.
Considering a random sequence of unit vectors r rfectly negative correlation is not obtained, because at each of the cross points at
ro, two of the vectors are in perfect alignment.
Considering a random sequence of unit vectors produces a clearly different result.
quenc

zero, two of the vectors are in perfect alignment.
Considering a random sequence of unit vectors produces a clearly different result.
Sequences of unit vectors were created by selecting random samples from a Fisher
distrib Considering a random sequence of unit vectors produces a clearly different result.
Sequences of unit vectors were created by selecting random samples from a Fisher
distribution with a specified mean and dispersion. The res Sequences of unit vectors were created by selecting random samples from a Fisher distribution with a specified mean and dispersion. The resulting autocorrelation functions are shown in figure 2 obtained from both methods distribution with a specified mean and dispersion. The resulting autocorrelation functions are shown in figure 2 obtained from both methods described above. In each case, the coefficient drops from a value of 1 to 0 in on tions are shown in figure 2 obtained from both methods described above. In each case, the coefficient drops from a value of 1 to 0 in only one offset (lag). This agrees with the interpretation that each direction of our s the coefficient drops from a value of 1 to
the interpretation that each direction of α
of the vector immediately preceding it.
If the series of random unit vectors is so If the vector immediately preceding it.
If the series of random unit vectors is smoothed so that each direction is dependent
on the preceding directions to some extent, very different results are obtained. The

of the vector immediately preceding it.
If the series of random unit vectors is smoothed so that each direction is dependent
upon the preceding directions to some extent, very different results are obtained. The
autocorrel If the series of random unit vectors is smoothed so that each direction is dependent
upon the preceding directions to some extent, very different results are obtained. The
autocorrelation functions determined using both th upon the preceding directions to some extent, very different results are obtained. The
autocorrelation functions determined using both the dot-product and the rotation-
matrix methods for a random sequence of unit vectors autocorrelation functions determined using both the dot-product and the rotation-
matrix methods for a random sequence of unit vectors that is smoothed over 5 and
10 points are also shown in figure 2. In each case, the ini matrix methods for a random sequence of unit vectors that is smoothed over 5 and 10 points are also shown in figure 2. In each case, the initial slope of the autocorrelation function becomes more gentle as the smoothing in 10 points are also shown in figure 2. In each case, the initial slope of the autocor-
relation function becomes more gentle as the smoothing interval is increased. The
results from both methods also suggest that the slope relation function becomes more gentle as the smoothing interval is increased. The results from both methods also suggest that the slope and intersection of the initial part of the function may be used to determine the memo results from both methods also suggest that the slope and intersection of the initial
part of the function may be used to determine the memory or dependence of the
vector sequence. This suggests that these methods may be u part of the function may be used to determine the memory vector sequence. This suggests that these methods may be used stratigraphic intervals in which secular variation is recorded.
In order to assess how well these metho vector sequence. This suggests that these methods may be useful in examining the stratigraphic intervals in which secular variation is recorded.
In order to assess how well these methods might work on real datasets, I sele

stratigraphic intervals in which secular variation is recorded.
In order to assess how well these methods might work on real datasets, I selected
two palaeomagnetic records that, in a sense, represent end members in terms In order to assess how well these methods might work on real datasets, I selected
two palaeomagnetic records that, in a sense, represent end members in terms of how
well they might be expected to record secular variation. two palaeomagnetic records that, in a sense, represent end members in terms of how
well they might be expected to record secular variation. For the first case, we examine
the secular variation record obtained from Fish Lak well they might be expected to record secular variation. For the first case, we examine
the secular variation record obtained from Fish Lake, Oregon (Hanna $\&$ Verosub
1989). This record was chosen because it documents l

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Figure 3. (*Cont.*) The results from the cumulative dispersion method (*b*) and from the autocorrelation methods obtained using both the dot-product method and the rotation-matrix method (c) . All three results appear to relation methods obtained using both the dot-product method and the rotation-matrix method (c) . All three results appear to be successful in indicating the nature of the smoothly varying secular variation record. (c) . All three results appear to be successful in indicating the nature of the smoothly varying

secular variation record.
that were recorded over time-scales of hundreds of years. The second example is the
record obtained from deep-sea core K78019. These data were obtained from very that were recorded over time-scales of hundreds of years. The second example is the record obtained from deep-sea core K78019. These data were obtained from very slowly accumulating sediments (10 m Myr^{-1}) that are not that were recorded over time-scales of hundreds of years. The second example is the record obtained from deep-sea core K78019. These data were obtained from very slowly accumulating sediments (10 m Myr^{-1}) that are not record obtained from deep-sea co
slowly accumulating sediments (1
secular variation in much detail. *Phil. Trans. R. Soc. Lond.* A (2000)

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mo. of samples included in the average
Figure 4. Results obtained from a much lower resolution record (a) obtained from deep sea
core K78019 (Thever *et al.* 1989). The palaeomagnetic directions shown here correspond to t Eigure 4. Results obtained from a much lower resolution record (a) obtained from deep sea
core K78019 (Theyer *et al.* 1989). The palaeomagnetic directions shown here correspond to the
interval of full reverse-polarity tha core K78019 (Theyer *et al.* 1989). The palaeomagnetic directions shown here correspond to the interval of full reverse-polarity that was used in this analysis. The results from the cumulative core K78019 (Theyer *et al.* 1989). The palaeomagnetic directions shown here correspond to the interval of full reverse-polarity that was used in this analysis. The results from the cumulative dispersion method (*b*) are interval of full reverse-polarity
dispersion method (b) are plo
each incremental averaging.

% each incremental averaging.
The directional record obtained from Fish Lake is shown in figure 3a together with The directional record obtained from Fish Lake is shown in figure 3a together with
the results of cumulative dispersion and autocorrelation analysis. The results of the
cumulative dispersion analysis are plotted in figure The directional record obtained from Fish Lake is shown in figure $3a$ together with
the results of cumulative dispersion analysis are plotted in figure $3b$. Both the mean k values and
the variance about those means exhi the results of cumulative dispersion and autocorrelation analysis. The results of the cumulative dispersion analysis are plotted in figure 3b. Both the mean k values and the variance about those means exhibit gentle slo cumulative dispersion analysis are plotted in figure 3b. Both the mean k values and the variance about those means exhibit gentle slopes indicating that a significant interval must be averaged over before arriving at th the variance about those means exhibit gentle slopes indicating that a significant
interval must be averaged over before arriving at the k value of the total population.
The mean k values very gradually approach the m interval must be averaged over before arriving at the k value of the total population.
The mean k values very gradually approach the mean, making it difficult to determine a specific interval. The variance, however, s

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Figure 4. (*Cont.*) The autocorrelation results (*c*) using both the dot-product and the rotation-matrix method are similar to the cumulative dispersion results, showing a very rapid decrease to low values consistent with Figure 4. (*Cont.*) The autocorrelation results (*c*) using both the dot-product and the tion-matrix method are similar to the cumulative dispersion results, showing a very decrease to low values, consistent with a nearly

samples, indicating that when 10 or more samples are included in the mean, the variance about the mean k value becomes significantly smaller. The variance also samples, indicating that when 10 or more samples are included in the mean, the variance about the mean k value becomes significantly smaller. The variance also shows a marked change in slope at 40 samples. In this recor samples, indicating that when 10 or more samples are included in the mean, the variance about the mean k value becomes significantly smaller. The variance also shows a marked change in slope at 40 samples. In this recor variance about the mean k value becomes significantly smaller. The variance also shows a marked change in slope at 40 samples. In this record 40 samples correspond roughly to 1600 yr. When more than 40 samples are inclu shows a marked change in slope at 40 samples. In this record 40 samples correspond roughly to 1600 yr. When more than 40 samples are included in the mean, both the mean k is very stable and the variance about that estimate of k no longer decreases with increasing numbers of samples. An extremely smooth mean k is very stable and the variance about that estimate of k no longer decreases
with increasing numbers of samples. An extremely smooth record that is serially
correlated appears to exhibit a variation in mean k that

correlated appears to exhibit a variation in mean k that is only a function of n .
The autocorrelation results obtained using both methods described above are
shown in figure 3c. Both curves exhibit gentle initial slop The autocorrelation results obtained using both methods described above are shown in figure 3c. Both curves exhibit gentle initial slopes, reaching zero between 12 and 20 offsets. These values represent a measure of the i shown in figure 3c. Both curves exhibit gentle initial slopes, reaching zero between 12
and 20 offsets. These values represent a measure of the independence of the samples
in this sequence, suggesting that, on average, onl In this sequence, suggesting that, on average, only every 10th to 20th sample may
be considered to be independent. Therefore, the initial slopes of these plots indicate
the memory of the process to be significant in this r in this sequence, suggesting that, on average, only every 10th to 20th sample may
be considered to be independent. Therefore, the initial slopes of these plots indicate
the memory of the process to be significant in this r be considered to be independent. Therefore, the initial slopes of these plots indicate
the memory of the process to be significant in this record; a conclusion that agrees
with the observation that this is a smoothly varyi with the observation that this is a smoothly varying sequence of directions with
swavelengths of the order of $10{-}20$ cm.

In contrast to the Fish Lake record, the record obtained from deep-sea core K78019 (Theyer *et al*. 1989) visually appears to have recorded a much-diminished record of In contrast to the Fish Lake record, the record obtained from deep-sea core K78019 (Theyer *et al.* 1989) visually appears to have recorded a much-diminished record of secular variation, both in amplitude and wavelength ((Theyer *et al.* 1989) visually appears to have recorded a much-diminished record of secular variation, both in amplitude and wavelength (figure 4a). Cumulative dispersion analysis of this record produces a plot of mean k secular variation, both in amplitude and wavelength (figure $4a$). Cumulative dispersion analysis of this record produces a plot of mean k values that exhibits a very steep initial slope (figure $4b$). Only five success sion analysis of this record produces a plot of mean k values that exhibits a very steep initial slope (figure 4b). Only five successive samples, on average, need to be included in order to obtain a mean with the disper included in order to obtain a mean with the dispersion characteristic of the whole
record. This core was sampled at an interval of 0.5 cm. Given a sedimentation rate included in order to obtain a mean with the dispersion characteristic of the whole
record. This corre was sampled at an interval of 0.5 cm. Given a sedimentation rate
of 10 m Myr⁻¹, this corresponds to an interval of 50 record. This core was sampled at an interval of 0.5 cm. Given a sedimentation rate
of 10 m Myr^{-1} , this corresponds to an interval of 500 yr . The results from autocor-
relation analysis are even more dramatic. Bo of 10 m Myr⁻¹, this corresponds to an interval of 500 yr. The results from autocor-
relation analysis are even more dramatic. Both results show an initial drop to values
close to zero, indicating that this record cannot relation analysis are even more dramatic
close to zero, indicating that this record
of directions about a mean (figure $4c$). *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 5. Virtual geomagnetic pole paths of three records of the Cobb Mountain Subchron
obtained from ODP Sites 837, 838 and 839 in the Lau Basin in the central equatorial Pacific Figure 5. Virtual geomagnetic pole paths of three records of the Cobb Mountain Subchron
obtained from ODP Sites 837, 838 and 839 in the Lau Basin in the central equatorial Pacific.
Although the records were obtained from c Figure 5. Virtual geomagnetic pole paths of three records of the Cobb Mountain Subchron
obtained from ODP Sites 837, 838 and 839 in the Lau Basin in the central equatorial Pacific.
Although the records were obtained from c obtained from ODP S:
Although the records
between the records.

7. Application to transition records

As a test to see if these methods are useful in assessing the temporal resolution of polarity transition records, we analysed a set of records of the same polarity

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Figure 6. Autocorrelation results from the Lau Basin Cobb Mountain records obtained using the dot-product method. The initial slopes are much steeper at Sites 838 and 839, whereas the initial slope is significantly less st dot-product method. The initial slopes are much steeper at Sites 838 and 839, whereas the initial slope is significantly less steep for the record from Site 837. This indicates that the records from
Site 873 exhibit a greater degree of serial correlation in the full polarity interval and, therefore,
may be expected to Site 873 exhibit a greater degree of serial correlation in the full polarity interval and, therefore,

reversal that were obtained from different sedimentation rate sequences. For the first case, we consider the records of the Cobb Mountain Subchron obtained from the reversal that were obtained from different sedimentation rate sequences. For the first
case, we consider the records of the Cobb Mountain Subchron obtained from the
Lau Basin (Abrahamsen & Sager 1994). These records are in case, we consider the records of the Cobb Mountain Subchron obtained from the Lau Basin (Abrahamsen & Sager 1994). These records are interesting because the same geomagnetic feature is recorded at three different sites wit same geomagnetic feature is recorded at three different sites with slightly varying sedimentation rates. The major features of the transition records are remarkably similar, but some significant differences also exist (figure 5; see also Clement (2000)). sedimentation rates. The major features of the transition records are remarkably
similar, but some significant differences also exist (figure 5; see also Clement (2000)).
Are these differences a result of the different res similar, but some significant differences also exist (figure 5; see also Clement (2000)).
Are these differences a result of the different resolution with which the sediments
have recorded the field, or do they mean that th have recorded the field, or do they mean that the recording is simply unreliable on these scales?

The autocorrelation analysis is a useful way of addressing this question. In figure 6, these scales?
The autocorrelation analysis is a useful way of addressing this question. In figure 6,
the results from the analysis of the same corresponding reverse polarity interval
are shown from Sites 837, 838 and 839. The autocorrelation analysis is a useful way of addressing this question. In figure 6, the results from the analysis of the same corresponding reverse polarity interval are shown from Sites 837, 838 and 839. These results the results from the analysis of the same corresponding reverse polarity interval
are shown from Sites 837, 838 and 839. These results indicate that in general, the
records from Sites 838 and 839 exhibit shorter-scale memo Caracteristic interval for averaging out secular variation is approximately only three
cords from Sites 838 and 839 exhibit shorter-scale memory, suggesting that the
conditional for averaging out secular variation is appr to four samples. This is consistent with the somewhat noisy nature of these records. characteristic interval for averaging out secular variation is approximately only three
to four samples. This is consistent with the somewhat noisy nature of these records.
The results from Site 837, however, indicate a ch to four samples. This is consiste
The results from Site 837, how
that of the other two records.
These results are interesting b These results from Site 837, however, indicate a characteristic interval nearly twice
at of the other two records.
These results are interesting because the record from Site 837 exhibits considerably
pre-detail in the pola

that of the other two records.
These results are interesting because the record from Site 837 exhibits considerably
more detail in the polarity transition than the records from Sites 838 and 839. And
even between Sites 838 These results are interesting because the record from Site 837 exhibits considerably
more detail in the polarity transition than the records from Sites 838 and 839. And
even between Sites 838 and 839, these results mean th *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 7. Autocorrelation results obtained from the upper, full reverse-polarity interval just
above the Cobb Mountain at DSDP Site 609. This plot shows that both methods indicate Figure 7. Autocorrelation results obtained from the upper, full reverse-polarity interval just
above the Cobb Mountain at DSDP Site 609. This plot shows that both methods indicate
a high degree of serial correlation that i above the Cobb Mountain at DSDP Site 609. This plot shows that both methods indicate a high degree of serial correlation that is consistent with the interpretation that this record provides greater resolution of the polarity transitions bounding the Cobb Mountain Subchron.

provides greater resolution of the polarity transitions bounding the Cobb Mountain Subchron.

on intervals shorter than three to four samples should be considered to be due to the

differences in the temporal resolution of differences in the temporal resolution of these records.
For comparison, analysis of the same corresponding part of the full reverse polarity intervals shorter than three to four samples should be considered to be due to the ferences in the temporal resolution of these records.
For comparison, analysis of the same corresponding part of the full reverse polarity

differences in the temporal resolution of these records.
For comparison, analysis of the same corresponding part of the full reverse polarity
section of the Cobb Mountain record from Site 609 is shown in figure 7 (Clement For comparison, analysis of the same corresponding part of the full reverse polarity
section of the Cobb Mountain record from Site 609 is shown in figure 7 (Clement $\&$
Kent 1987). This record is from a core with almost section of the Cobb Mountain record from Site 609 is shown in figure 7 (Clement $\&$ Kent 1987). This record is from a core with almost an order of magnitude difference in sedimentation rate and visually appears to have r Kent 1987). This record is from a core with almost an order of magnitude difference in
sedimentation rate and visually appears to have recorded secular variation in greater
detail than the Lau Basin records. The autocorrel sedimentation rate and visually appears to have recorded secular variation in greater
detail than the Lau Basin records. The autocorrelation plots for this record indicate
that only every 20th–30th sample can be considered detail than the Lau Basin records. The autocorrelation plots for this record indicate
that only every 20th–30th sample can be considered to be independent. This inter-
val, therefore, probably corresponds to the characteri secular variation is recorded over. The greater length suggests that the corresponding val, therefore, probably corresponds to the characteristic stratigraphic intervase
cular variation is recorded over. The greater length suggests that the corresport
ransition record may be interpreted as a comparable incre ed as a comparable i
8. Discussion

8. Discussion
The tests of the three methods discussed above—using sequences of sinusoidally vary-
ing random, and progressively smoothed unit vectors—suggest that these methods The tests of the three methods discussed above—using sequences of sinusoidally vary-
ing, random, and progressively smoothed unit vectors—suggest that these methods
may be a useful tool in assessing the temporal resolution The tests of the three methods discussed above—using sequences of sinusoidally vary-
ing, random, and progressively smoothed unit vectors—suggest that these methods
may be a useful tool in assessing the temporal resolution ing, random, and progressively smoothed unit vectors—suggest that these methods may be a useful tool in assessing the temporal resolution provided by a palaeomagnetic record of field behaviour. This information will be of may be a useful tool in assessing the temporal resolution provided by a palaeomagnetic record of field behaviour. This information will be of use as we attempt to compile multiple records of the same reversals. It will be netic record of field behaviour. This information will be of use as we attempt to compile multiple records of the same reversals. It will be helpful to assess just how much of the differences between two records may be rel compile multiple records of the same reversals. It will be helpful to assess just how
much of the differences between two records may be related solely to the tempo-
ral resolution of the records and how much represents re much of the differences between two records may be related solely to the temporal resolution of the records and how much represents real differences in the field behaviour or the recording process.

For example, if the autocorrelation results indicate a random sequence during behaviour or the recording process.
For example, if the autocorrelation results indicate a random sequence during
full polarity, then that particular record should not be interpreted on the scales of
transitional field beh For example, if the autocorrelation results indicate a random sequence during full polarity, then that particular record should not be interpreted on the scales of transitional field behaviour. These methods, therefore, pr *Phil. Trans. R. Soc. Lond.* A (2000) **Phil.** *Phil. Trans. R. Soc. Lond.* A (2000)

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the question of whether a very rapid polarity change (i.e. between two adjacent samples) should be interpreted as an incredibly fast reversal. If the autocorrelation the question of whether a very rapid polarity change (i.e. between two adjacent
samples) should be interpreted as an incredibly fast reversal. If the autocorrelation
analysis indicates that the full polarity data are not s samples) should be interpreted as an incredibly fast reversal. If the autocorrelation analysis indicates that the full polarity data are not significantly different from a random selection of directions about a mean, then random selection of directions about a mean, then the interpretation of rapid reversal
is not warranted. However, if the autocorrelation analysis indicates that a significant random selection of directions about a mean, then the interpretation of rapid reversal
is not warranted. However, if the autocorrelation analysis indicates that a significant
interval is required to average out secular var is not warranted. However,
interval is required to ave:
interpreted to that scale.
The arguments used abo terval is required to average out secular variation, then field variations may be
terpreted to that scale.
The arguments used above assume that a smoothly varying field is a good indi-
tor that secular variation has been r

interpreted to that scale.
The arguments used above assume that a smoothly varying field is a good indicator that secular variation has been recorded. However, it is often argued that the remanence acquisition process in s The arguments used above assume that a smoothly varying field is a good indicator that secular variation has been recorded. However, it is often argued that the remanence acquisition process in sediments effectively smooth cator that secular variation has been recorded. However, it is often argued that the remanence acquisition process in sediments effectively smoothes the record of field behaviour. Therefore, one could interpret the initial remanence acquisition process in sediments effectively smoothes the record of field
behaviour. Therefore, one could interpret the initial slope in the autocorrelation plots
as an indicator of the extent of smoothing. This behaviour. Therefore, one could interpret the initial slope in the autocorrelation plots
as an indicator of the extent of smoothing. This can best be addressed by examin-
ing palaeomagnetic records that are considered to h as an indicator of the extent of smoothing. This can best be addressed by examining palaeomagnetic records that are considered to have recorded secular variation.
In this paper, the two end members have been examined, and ing palaeomagnetic records that are considered to have recorded secular variation.
In this paper, the two end members have been examined, and the more smoothly
varying record is the one that provides the better record of s In this paper, the two end members have been examined, and the more smoothly
varying record is the one that provides the better record of secular variation. The
low sedimentation rate record is an example of a case where t varying record is the one that provides the better record of secular variation. The
low sedimentation rate record is an example of a case where the secular variation
has been effectively completely averaged out, and the va low sedimentation rate record is an example of a case where the secular variation
has been effectively completely averaged out, and the variation between adjacent
samples probably results from random errors about the mean s been effectively completely averaged out, and the variation between adjacent
mples probably results from random errors about the mean smoothed direction.
A major question that remains in this approach is that of separati

samples probably results from random errors about the mean smoothed direction.
A major question that remains in this approach is that of separating the memory of
of the palaeomagnetic recorder (how smoothed the signal is) A major question that remains in this approach is that of separating the memory of the palaeomagnetic recorder (how smoothed the signal is) from the memory of the geomagnetic field. In this paper, the assumption is made th of the palaeomagnetic recorder (how smoothed the signal is) from the memory of
the geomagnetic field. In this paper, the assumption is made that the geomagnetic
field varies on shorter time-scales than can possibly be reco the geomagnetic field. In this paper, the assumption is made that the geomagnetic
field varies on shorter time-scales than can possibly be recorded by a sedimentary
recorder. By making this assumption, the methods outlined field varies on shorter time-scales than can possibly be recorded by a sedimentary recorder. By making this assumption, the methods outlined here can provide a way of examining the lower limit to which a palaeomagnetic rec recorder. By making this assumption
of examining the lower limit to which
anges in the geomagnetic field.

9. Conclusions

9. Conclusions
The methods described in this paper provide a mechanism to assess the temporal res-
olution of palaeomagnetic records without making any assumptions about the nature The methods described in this paper provide a mechanism to assess the temporal resolution of palaeomagnetic records without making any assumptions about the nature of the field behaviour in detail. These methods are also i **TYSICAL
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IENCES** The methods described in this paper provide a mechanism to assess the temporal res-
olution of palaeomagnetic records without making any assumptions about the nature
of the field behaviour in detail. These methods are also olution of palaeomagnetic records without making any assumptions about the nature
of the field behaviour in detail. These methods are also internal to each record. The
cumulative dispersion method provides a way (that will of the field behaviour in detail. These methods are also internal to each record. The cumulative dispersion method provides a way (that will be familiar to most palaeo-
magnetists) to determine, on average, the stratigraph

cumulative dispersion method provides a way (that will be familiar to most palaeo-
magnetists) to determine, on average, the stratigraphic interval that must be included
in order to average out secular variation. This may magnetists) to determine, on average, the stratigraphic interval that must be included
in order to average out secular variation. This may be interpreted as the character-
istic interval over which secular variation is rec in order to average out secular variation. This may be interpreted as the characteristic interval over which secular variation is recorded. As such, it provides important constraints on the extent to which the record shoul istic interval over which secular variation is recorded. As such, it provides important constraints on the extent to which the record should be interpreted in terms of temporal resolution. The other two methods involve cal constraints on the extent to which the record should be interpreted in terms of
temporal resolution. The other two methods involve calculating the autocorrelation
function for a series of unit vectors. Two previously publi temporal resolution. The other two methods involve calculating the autocorrelation
function for a series of unit vectors. Two previously published approaches to vector
correlation were tried in the autocorrelation analyses - function for a series of unit vectors. Two previously published approaches to vector \Box correlation were tried in the autocorrelation analyses. As in the case of traditional \Box autocorrelation of a scalar series, th correlation were tried in the autocorrelation analyses. As in the case of traditional autocorrelation of a scalar series, the initial slope of the plot may be interpreted in terms of the memory of the process. This, in tur autocorrelation of a scalar series, the initial slope of the plot may be interpreted in terms of the memory of the process. This, in turn, may also be interpreted in terms of a characteristic interval defined by the record Further testing of the process. This, in turn, may also be interpreted in terms
a characteristic interval defined by the recorded secular variation in that record.
Further testing of this approach using records that may be $\mathbf S$

of a characteristic interval defined by the recorded secular variation in that record.
Further testing of this approach using records that may be tested using independent means should help determine just how suitable it is Further testing of this approach using records that may be tested using independent means should help determine just how suitable it is. For now, this method appears to provide a valuable quantitative measure of the tempor dent means should help determ
appears to provide a valuable que
a record should be interpreted.

The cord should be interpreted.
Comments by Nicholas Teanby significantly improved this manuscript. This work was supported
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